PEPPERDINE UNIVERSITY

Seaver College

NATURAL SCIENCE DIVISION

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The Tooma Undergraduate Research Fellowship Program:

I am pleased to announce the publication of "A Calculational Deductive System for Linear Temporal Logic" in the Association for Computing Machinery's (ACM) journal *Computing Surveys*. The research for the publication was funded in part by the Tooma Undergraduate Research Fellowship Program, for which we are truly grateful. Attached is the first page of the article with the acknowledgment.

ACM is the premier international professional association for computer science. And, the *Computing Surveys* journal is its highest impact journal.

It has been a full ten years since we received support from the Tooma Foundation, and therein lies a story. Your recipient, David Vega, was an undergraduate student of mine in a course titled Programming Paradigms that teaches the theory and practice of alternative programming languages. One of these programming languages is analyzed by a system of logic called linear temporal logic, which we study in the course. Unfortunately, the treatment in the literature was at the graduate level and was fragmented.

David and I had the idea to apply a new deductive system to the logic. Little did we know how extensive an undertaking this would turn out to be. We did present our preliminary work at Seaver College's undergraduate research symposium those many years ago.

David graduated, received his graduate degree in computer science, and now works in the aerospace industry. We kept in touch and continued to collaborate on the project. In the meantime, my course lectures were video recorded and published online. Scott Staley, a researcher at Ford Motor Company Research Labs, discovered our work from the recordings, began to contribute to the research, and eventually became our third author.

The final work consists of 256 logic theorems, many of which have not previously appeared in the literature. One reviewer called it a "tour de force."

Thank you for the initial funding that eventually led to this publication.

Sincerely yours,

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A Calculational Deductive System for Linear Temporal Logic

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This article surveys the linear temporal logic (LTL) literature and presents all the LTL theorems from the survey, plus many new ones, in a calculational deductive system. Calculational deductive systems, developed by Dijkstra and Scholten and extended by Gries and Schneider, are based on only four inference rules—Substitution, Leibniz, Equanimity, and Transitivity. Inference rules in the older Hilbert-style systems, notably modus ponens, appear as theorems in this calculational deductive system. This article extends the calculational deductive system of Gries and Schneider to LTL, using only the same four inference rules. Although space limitations preclude giving a proof of every theorem in this article, every theorem has been proved with calculational logic.

CCS Concepts: • **Theory of computation** → **Modal and temporal logics**;

Additional Key Words and Phrases: Calculational logic, equational logic, linear temporal logic

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1 INTRODUCTION

Linear temporal logic (LTL) has application to proof of correctness for concurrent programs. Many concurrent programs, such as operating systems and embedded systems that control physical equipment, are nonterminating by design. Consequently, proof techniques that depend on proving the correctness of postconditions on program termination do not apply. LTL, however, can be used to prove desirable program traits such as freedom from deadlock.

Most treatments of LTL consist of cursory introductions in one or two chapters of graduate-level textbooks [2, 20, 21, 24]. While many LTL theorems are common in the different treatments, each treatment has theorems that are unique to it. This survey is a comprehensive collection of all the LTL theorems that we have found in the literature, together with many new theorems, all of which are presented in an axiomatic logic system. It serves as an introduction to LTL and should be accessible with a prerequisite only of the standard propositional and predicate logic at the undergraduate level.

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