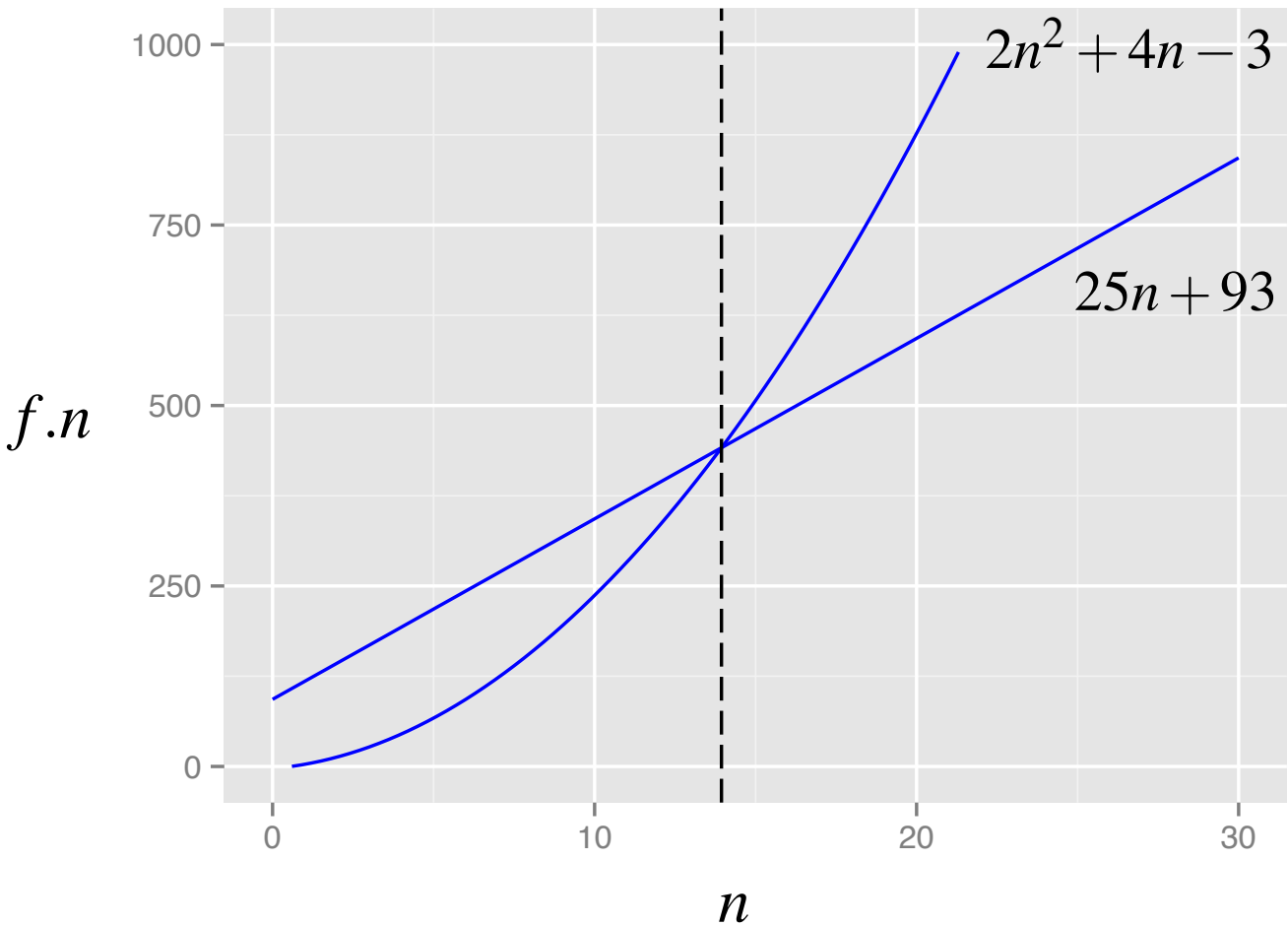
Asymptotic Analysis

- Algorithm A: $f.n = 25n + 93$
- Algorithm B: $f.n = 2n^2 + 4n - 3$

Which is better?

Figure 3.5

- Algorithm A: $f.n = 25n + 93$
- Algorithm B: $f.n = 2n^2 + 4n - 3$



Definition of asymptotic upper bound: For a given function $g.n$, $O(g.n)$, pronounced "big-oh of g of n", is the set of functions

$$\{f.n \mid (\exists c, n_0 \mid c > 0 \wedge n_0 > 0 : (\forall n \mid n \geq n_0 : 0 \leq f.n \leq c \cdot g.n)\ )\}\,.$$

O-notation: $f.n = O(g.n)$ means function $f.n$ is in the set $O(g.n)$.

$$25n + 93 = O(n)$$

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Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \leq cn \quad \text{for } n \geq n_0$.

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$25n + 93$

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Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \leq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 25n + 93 \\ \leq \quad & \langle \text{Replacing 93 with a larger value, provided } n \geq 93 \rangle \end{aligned}$$

$$25n + 93 = O(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \leq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 25n + 93 \\ \leq \quad & \langle \text{Replacing 93 with a larger value, provided } n \geq 93 \rangle \\ & 25n + n \end{aligned}$$

$$25n + 93 = O(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \leq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 25n + 93 \\
\leq \quad & \langle \text{Replacing 93 with a larger value, provided } n \geq 93 \rangle \\
& 25n + n \\
= \quad & \langle \text{Math} \rangle
\end{aligned}$$

$$25n + 93 = O(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \leq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 25n + 93 \\
\leq \quad & \langle \text{Replacing 93 with a larger value, provided } n \geq 93 \rangle \\
& 25n + n \\
= \quad & \langle \text{Math} \rangle \\
& 26n
\end{aligned}$$

$$25n + 93 = O(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \leq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 25n + 93 \\
\leq \quad & \langle \text{Replacing 93 with a larger value, provided } n \geq 93 \rangle \\
& 25n + n \\
= \quad & \langle \text{Math} \rangle \\
& 26n \\
= \quad & \langle \text{Provided } c = 26 \rangle
\end{aligned}$$

$$25n + 93 = O(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \leq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 25n + 93 \\
\leq \quad & \langle \text{Replacing 93 with a larger value, provided } n \geq 93 \rangle \\
& 25n + n \\
= \quad & \langle \text{Math} \rangle \\
& 26n \\
= \quad & \langle \text{Provided } c = 26 \rangle \\
& cn
\end{aligned}$$

$$25n + 93 = O(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \leq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 25n + 93 \\
\leq \quad & \langle \text{Replacing 93 with a larger value, provided } n \geq 93 \rangle \\
& 25n + n \\
= \quad & \langle \text{Math} \rangle \\
& 26n \\
= \quad & \langle \text{Provided } c = 26 \rangle \\
& cn
\end{aligned}$$

So, $c = 26, n_0 = 93$. ∎

$$2n^2 + 4n - 3 = O(n^2)$$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$2n^2 + 4n - 3$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 2n^2 + 4n - 3 \\ \leq \quad & \langle \text{Eliminating a negative value} \rangle \end{aligned}$$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 2n^2 + 4n - 3 \\ \leq \quad & \langle \text{Eliminating a negative value} \rangle \\ & 2n^2 + 4n \end{aligned}$$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\leq \quad & \langle \text{Eliminating a negative value} \rangle \\
& 2n^2 + 4n \\
\leq \quad & \langle \text{Replacing } 4n \text{ with a larger value, provided } n \geq 1 \rangle
\end{aligned}$$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\leq \quad & \langle \text{Eliminating a negative value} \rangle \\
& 2n^2 + 4n \\
\leq \quad & \langle \text{Replacing } 4n \text{ with a larger value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n \cdot n
\end{aligned}$$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\leq \quad & \langle \text{Eliminating a negative value} \rangle \\
& 2n^2 + 4n \\
\leq \quad & \langle \text{Replacing } 4n \text{ with a larger value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n \cdot n \\
= \quad & \langle \text{Math} \rangle
\end{aligned}$$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\leq \quad & \langle \text{Eliminating a negative value} \rangle \\
& 2n^2 + 4n \\
\leq \quad & \langle \text{Replacing } 4n \text{ with a larger value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n \cdot n \\
= \quad & \langle \text{Math} \rangle \\
& 6n^2
\end{aligned}$$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\leq \quad & \langle \text{Eliminating a negative value} \rangle \\
& 2n^2 + 4n \\
\leq \quad & \langle \text{Replacing } 4n \text{ with a larger value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n \cdot n \\
= \quad & \langle \text{Math} \rangle \\
& 6n^2 \\
= \quad & \langle \text{Provided } c = 6 \rangle
\end{aligned}$$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$
\begin{aligned}
& 2n^2 + 4n - 3 \\
\leq \quad & \langle \text{Eliminating a negative value} \rangle \\
& 2n^2 + 4n \\
\leq \quad & \langle \text{Replacing } 4n \text{ with a larger value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n \cdot n \\
= \quad & \langle \text{Math} \rangle \\
& 6n^2 \\
= \quad & \langle \text{Provided } c = 6 \rangle \\
& cn^2
\end{aligned}
$$

$$2n^2 + 4n - 3 = O(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \leq cn^2 \quad \text{for } n \geq n_0$.

$$
\begin{aligned}
& 2n^2 + 4n - 3 \\
\leq \quad & \langle \text{Eliminating a negative value} \rangle \\
& 2n^2 + 4n \\
\leq \quad & \langle \text{Replacing } 4n \text{ with a larger value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n \cdot n \\
= \quad & \langle \text{Math} \rangle \\
& 6n^2 \\
= \quad & \langle \text{Provided } c = 6 \rangle \\
& cn^2
\end{aligned}
$$

So, $c = 6, n_0 = 1$. ∎

Definition of asymptotic lower bound: For a given function $g.n$, $\Omega(g.n)$, pronounced "big-omega of g of n", is the set of functions

$$\{f.n \mid (\exists c, n_0 \mid c > 0 \wedge n_0 > 0 : (\forall n \mid n \geq n_0 : 0 \leq c \cdot g.n \leq f.n)\ )\}.$$

Ω-notation: $f.n = \Omega(g.n)$ means function $f.n$ is in the set $\Omega(g.n)$.

$$25n + 93 = \Omega(n)$$

$$25n + 93 = \Omega(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \geq cn \quad \text{for } n \geq n_0$.

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$25n + 93$

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Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \geq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 25n + 93 \\ \geq \quad & \langle \text{Eliminating a positive value} \rangle \end{aligned}$$

$$25n + 93 = \Omega(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \geq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 25n + 93 \\ \geq \quad & \langle \text{Eliminating a positive value} \rangle \\ & 25n \end{aligned}$$

$$25n + 93 = \Omega(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \geq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 25n + 93 \\
\geq \quad & \langle \text{Eliminating a positive value} \rangle \\
& 25n \\
= \quad & \langle \text{Provided } c = 25 \rangle
\end{aligned}$$

$$25n + 93 = \Omega(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \geq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 25n + 93 \\
\geq \quad & \langle \text{Eliminating a positive value} \rangle \\
& 25n \\
= \quad & \langle \text{Provided } c = 25 \rangle \\
& cn
\end{aligned}$$

$$25n + 93 = \Omega(n)$$

Proof: Must prove that there exist positive constants c, n_0 such that $25n + 93 \geq cn \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 25n + 93 \\
\geq \quad & \langle \text{Eliminating a positive value} \rangle \\
& 25n \\
= \quad & \langle \text{Provided } c = 25 \rangle \\
& cn
\end{aligned}$$

So, $c = 25$, and any positive n_0 is possible, say $n_0 = 1$. ∎

$$2n^2 + 4n - 3 = \Omega(n^2)$$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$2n^2 + 4n - 3$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 2n^2 + 4n - 3 \\ \geq \quad & \langle \text{Increasing a negative value, provided } n \geq 1 \rangle \end{aligned}$$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 2n^2 + 4n - 3 \\ \geq \quad & \langle \text{Increasing a negative value, provided } n \geq 1 \rangle \\ & 2n^2 + 4n - 3n \end{aligned}$$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\geq \quad & \langle \text{Increasing a negative value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n - 3n \\
= \quad & \langle \text{Math} \rangle
\end{aligned}$$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\geq \quad & \langle \text{Increasing a negative value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n - 3n \\
= \quad & \langle \text{Math} \rangle \\
& 2n^2 + n
\end{aligned}$$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\geq \quad & \langle \text{Increasing a negative value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n - 3n \\
= \quad & \langle \text{Math} \rangle \\
& 2n^2 + n \\
\geq \quad & \langle \text{Eliminating a positive value, provided } n \geq 0 \rangle
\end{aligned}$$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\geq \quad & \langle \text{Increasing a negative value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n - 3n \\
= \quad & \langle \text{Math} \rangle \\
& 2n^2 + n \\
\geq \quad & \langle \text{Eliminating a positive value, provided } n \geq 0 \rangle \\
& 2n^2
\end{aligned}$$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\geq \quad & \langle \text{Increasing a negative value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n - 3n \\
= \quad & \langle \text{Math} \rangle \\
& 2n^2 + n \\
\geq \quad & \langle \text{Eliminating a positive value, provided } n \geq 0 \rangle \\
& 2n^2 \\
= \quad & \langle \text{Provided } c = 2 \rangle
\end{aligned}$$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\geq \quad & \langle \text{Increasing a negative value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n - 3n \\
= \quad & \langle \text{Math} \rangle \\
& 2n^2 + n \\
\geq \quad & \langle \text{Eliminating a positive value, provided } n \geq 0 \rangle \\
& 2n^2 \\
= \quad & \langle \text{Provided } c = 2 \rangle \\
& cn^2
\end{aligned}$$

$$2n^2 + 4n - 3 = \Omega(n^2)$$

Proof: Must prove that there exist positive constants c, n_0 such that $2n^2 + 4n - 3 \geq cn^2 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 2n^2 + 4n - 3 \\
\geq \quad & \langle \text{Increasing a negative value, provided } n \geq 1 \rangle \\
& 2n^2 + 4n - 3n \\
= \quad & \langle \text{Math} \rangle \\
& 2n^2 + n \\
\geq \quad & \langle \text{Eliminating a positive value, provided } n \geq 0 \rangle \\
& 2n^2 \\
= \quad & \langle \text{Provided } c = 2 \rangle \\
& cn^2
\end{aligned}$$

So, $c = 2, n_0 = 1$. ∎

$$6n^3 - 7n = \Omega(n^3)$$

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Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$6n^3 - 7n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$6n^3 - 7n$$

$$6n^3 - 7n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 6n^3 - 7n \\ \geq \quad & \langle \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \rangle \\ & 5n^3 \end{aligned}$$

$$6n^3 - 7n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 6n^3 - 7n \\ \geq \quad & \langle 6n^3 - 7n \geq 5n^3, \qquad \rangle \\ & 5n^3 \end{aligned}$$

$$6n^3 - 7n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 6n^3 - 7n \\ \geq \quad & \langle 6n^3 - 7n \geq 5n^3,\ n^3 \geq 7n, \qquad \rangle \\ & 5n^3 \end{aligned}$$

$$6n^3 - 7n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 6n^3 - 7n \\ \geq \quad & \langle 6n^3 - 7n \geq 5n^3,\ n^3 \geq 7n,\ n^2 \geq 7, \qquad \rangle \\ & 5n^3 \end{aligned}$$

$$6n^3 - 7n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & 6n^3 - 7n \\ \geq \quad & \langle 6n^3 - 7n \geq 5n^3,\ n^3 \geq 7n,\ n^2 \geq 7,\ n \geq \sqrt{7}\ \rangle \\ & 5n^3 \end{aligned}$$

$$6n^3 - 7n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 6n^3 - 7n \\
\geq \quad & \langle 6n^3 - 7n \geq 5n^3,\ n^3 \geq 7n,\ n^2 \geq 7,\ n \geq \sqrt{7}\ \rangle \\
& 5n^3 \\
= \quad & \langle \text{Provided } c = 5 \rangle
\end{aligned}$$

$$6n^3 - 7n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 6n^3 - 7n \\
\geq \quad & \langle 6n^3 - 7n \geq 5n^3,\ n^3 \geq 7n,\ n^2 \geq 7,\ n \geq \sqrt{7}\ \rangle \\
& 5n^3 \\
= \quad & \langle \text{Provided } c = 5 \rangle \\
& cn^3
\end{aligned}$$

$$6n^3 - 7n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $6n^3 - 7n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& 6n^3 - 7n \\
\geq \quad & \langle 6n^3 - 7n \geq 5n^3,\ n^3 \geq 7n,\ n^2 \geq 7,\ n \geq \sqrt{7}\ \rangle \\
& 5n^3 \\
= \quad & \langle \text{Provided } c = 5 \rangle \\
& cn^3
\end{aligned}$$

So, $c = 5, n_0 = \sqrt{7}$. ∎

$$n^3 - 5n = \Omega(n^3)$$

$$n^3 - 5n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $n^3 - 5n \geq cn^3 \quad \text{for } n \geq n_0$.

$$n^3 - 5n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $n^3 - 5n \geq cn^3 \quad \text{for } n \geq n_0$.

$$n^3 - 5n$$

$$n^3 - 5n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $n^3 - 5n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{array}{ll} & n^3 - 5n \\ \geq & \langle \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \rangle \\ & 0.5n^3 \end{array}$$

$$n^3 - 5n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $n^3 - 5n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & n^3 - 5n \\ \geq \quad & \langle n^3 - 5n \geq 0.5n^3, \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \rangle \\ & 0.5n^3 \end{aligned}$$

$$n^3 - 5n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $n^3 - 5n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & n^3 - 5n \\ \geq \quad & \langle n^3 - 5n \geq 0.5n^3,\ 0.5n^3 \geq 5n, \qquad \rangle \\ & 0.5n^3 \end{aligned}$$

$$n^3 - 5n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $n^3 - 5n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & n^3 - 5n \\ \geq \quad & \langle n^3 - 5n \geq 0.5n^3,\ 0.5n^3 \geq 5n,\ n^2 \geq 10, \qquad \rangle \\ & 0.5n^3 \end{aligned}$$

$$n^3 - 5n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $n^3 - 5n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned} & n^3 - 5n \\ \geq \quad & \langle n^3 - 5n \geq 0.5n^3,\ 0.5n^3 \geq 5n,\ n^2 \geq 10,\ n \geq \sqrt{10}\ \rangle \\ & 0.5n^3 \end{aligned}$$

$$n^3 - 5n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $n^3 - 5n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& n^3 - 5n \\
\geq \quad & \langle n^3 - 5n \geq 0.5n^3,\ 0.5n^3 \geq 5n,\ n^2 \geq 10,\ n \geq \sqrt{10}\ \rangle \\
& 0.5n^3 \\
= \quad & \langle \text{Provided } c = 0.5 \rangle
\end{aligned}$$

$$n^3 - 5n = \Omega(n^3)$$

Proof: Must prove that there exist positive constants c, n_0 such that $n^3 - 5n \geq cn^3 \quad \text{for } n \geq n_0$.

$$\begin{aligned}
& n^3 - 5n \\
\geq \quad & \langle n^3 - 5n \geq 0.5n^3,\ 0.5n^3 \geq 5n,\ n^2 \geq 10,\ n \geq \sqrt{10}\ \rangle \\
& 0.5n^3 \\
= \quad & \langle \text{Provided } c = 0.5 \rangle \\
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So, $c = 0.5, n_0 = \sqrt{10}$. ∎

Definition of asymptotic tight bound: For a given function $g.n$, $\Theta(g.n)$, pronounced "big-theta of g of n", is the set of functions

$$\{f.n \mid (\exists c_1, c_2, n_0 \mid c_1 > 0 \wedge c_2 > 0 \wedge n_0 > 0 : \\ (\forall n \mid n \geq n_0 : 0 \leq c_1 \cdot g.n \leq f.n \leq c_2 \cdot g.n)\ )\}.$$

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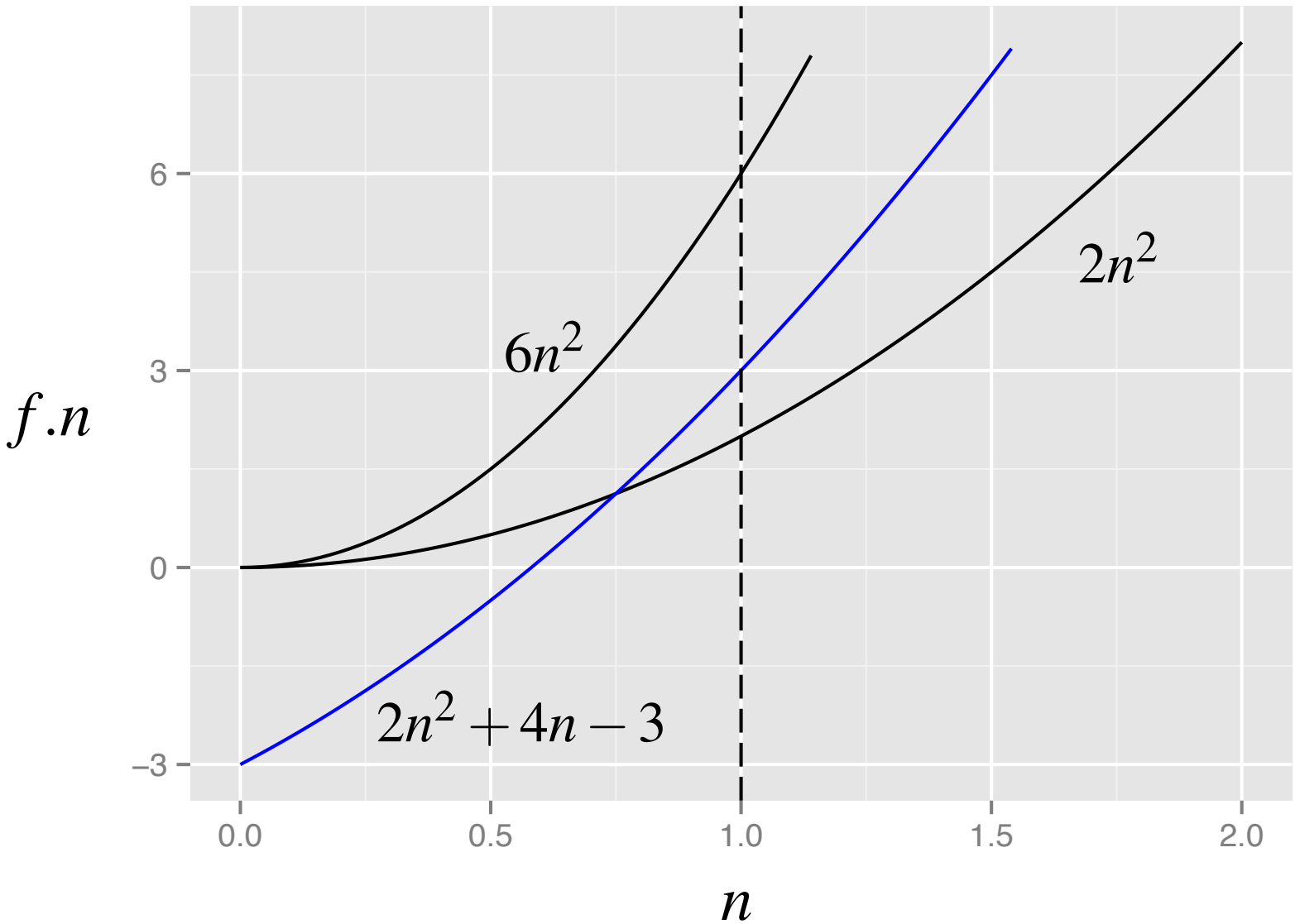
$$25n + 93 = \Theta(n)$$

$$2n^2 + 4n - 3 = \Theta(n^2)$$

Interpretation of big theta



Polynomial bound: Define an *asymptotically positive polynomial* $p.n$ *of degree* d to be

$$p.n = (\Sigma\, i \mid 0 \leq i \leq d : a_i n^i)$$

where the constants $a_0, a_1, \ldots, a_d$ are the *coefficients* of the polynomial and $a_d > 0$. Then $p.n = \Theta(n^d)$.